Exercise 37

The gas law for an ideal gas at absolute temperature T (in kelvins), pressure P (in atmospheres), and volume V (in liters) is $PV = nRT$, where n is the number of moles of the gas and $R = 0.0821$ is the gas constant. Suppose that, at a certain instant, $P = 8.0$ atm and is increasing at a rate of 0.10 atm/min and $V = 10$ L and is decreasing at a rate of 0.15 L/min. Find the rate of change of T with respect to time at that instant if $n = 10$ mol.

Solution

Suppose the ideal gas law is true.

$$PV = nRT$$

Solve the equation for T.

$$T = \frac{PV}{nR}$$

Take the derivative of T with respect to t.

$$\begin{aligned}\frac{dT}{dt} &= \frac{d}{dt}\left(\frac{PV}{nR}\right) \\ &= \frac{1}{nR}\frac{d}{dt}(PV) \\ &= \frac{1}{nR}\left(\frac{dP}{dt}V + P\frac{dV}{dt}\right)\end{aligned}$$

Use the fact that $R = 0.0821\ \text{L}\cdot\text{atm}/(\text{mol}\cdot\text{K})$, $P = 8.0$ atm, $dP/dt = 0.10$ atm/min, $V = 10$ L, $dV/dt = -0.15$ L/min, and $n = 10$ mol.

$$\begin{aligned}\left.\frac{dT}{dt}\right|_{t=\text{instant}} &= \frac{1}{(10\ \text{mol})\left(0.0821\ \frac{\text{L}\cdot\text{atm}}{\text{mol}\cdot\text{K}}\right)}\left[\left(0.10\ \frac{\text{atm}}{\text{min}}\right)(10\ \text{L}) + (8.0\ \text{atm})\left(-0.15\ \frac{\text{L}}{\text{min}}\right)\right] \\ &\approx -0.243605\ \frac{\text{K}}{\text{min}}\end{aligned}$$